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Stationary spacetime from intersecting M-branes

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Abstract

We study a stationary 'black' brane in M/superstring theory. Assuming BPS-type relations between the first-order derivatives of metric functions, we present general stationary black brane solutions with a time-like Killing vector for the Einstein equations in D-dimensions. The solutions are given by a few independent harmonic equations (and plus the Poisson equation). General solutions are constructed by superposition of a complete set of those harmonic functions. Using the hyperspherical coordinate system, we explicitly give the solutions in 11-dimensional M theory for the case with M2$\perp$ M2$\perp$ M2 intersecting branes. Compactifying these solutions into five dimensions, we show that these solutions include the supersymmetric black ring solution. We prove that the solutions preserve the 1/8 supersymmetry if the gravi-electromagnetic field $\mathcal{F}_{ij}$, which is a rotational part of gravity, is self-dual, or to add the additional constraint for the integral constants.

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1. Introduction

Black holes are now one of the most important subjects in string theory. The Beckenstein-Hawking black hole entropy of an extreme black hole is obtained in string theory by statistical counting of the corresponding microscopic states [1]. While, we have found several interesting black hole solutions in supergravity theories [2–7], which are obtained as an effective theory of a superstring model in a low energy limit. We also know black hole solutions in a higher-dimensional spacetime [8, 9], which play a key role in a unified theory such as string theory. In higher dimensions, because there is no uniqueness theorem of black holes [10–12], we have a variety of 'black' objects such as a black brane [13–16]. One of the most remarkable solutions is a black ring, whose horizon has a topology of $S^1 \times S^2$ [17].

Among such 'black' objects, supersymmetric ones are very important. The black hole solutions in a supergravity include the higher-order effects of a string coupling constant, although these are solutions in a low energy limit. On the other hand, the counting of states of corresponding branes is performed at the lowest order of a string coupling. The results of these two calculations need not coincide each other. However, if there is supersymmetry, these should be the same because the numbers of dynamical freedom cannot be different in these BPS representations. Therefore, supersymmetric black hole (or black ring) solutions are often discussed in many literatures [18–22].

The classification of supersymmetric solutions in minimal $\mathcal{N} = 2$ supergravity in $D = 4$ was first performed by a time-like or null Killing spinor [23]. Recently, solutions in minimal $\mathcal{N} = 1$ supergravity in $D = 5$ have been classified into two classes by use of G-structure analysis [24]. The six-dimensional minimal supergravity has also been discussed [25].

However, the fundamental theory is constructed in either ten or eleven dimensions. When we discuss the entropy of black holes, we have to show the relation between those supersymmetric black holes and more fundamental 'black' branes in either $D = 10$ or 11, from which we obtain 'black' holes (or rings) via compactification. The entropy is microscopically described by the charges of branes [26]. A supersymmetric rotating solution is obtained by compactification from M or type II supergravity [27]. The supersymmetric rotating black ring solution is found [28, 29]. Such solutions are obtained also in lower dimensions. These solutions are in fact new classes of rotating solutions in four- or five-dimensional supergravity. The existence of such solutions suggests that the uniqueness theorem of black holes is no longer valid even in supersymmetric spacetime if the dimension is five or higher [30]. Thus we may need to construct more generic 'black' brane solutions in the fundamental theory and the black holes by some compactification. M-theory is the best candidate for such a unified theory. Since its low energy limit coincides with the 11-dimensional supergravity, it provides a natural framework to study 'black' brane or BPS brane solutions.

In this paper we study a class of intersecting brane solutions in D-dimensions with a $(d-1)$-dimensional transverse conformally flat space. We start with a generic form of the metric and solve the field equations of the supergravity (the Einstein equations and the equations for form fields). Assuming the intersection rule for the intersecting branes, which is the same as that derived in a spherically symmetric case [31], we derive the equations for each metric.

2. Basic equations for a stationary spacetime with branes

We first present the basic equations for a stationary spacetime with intersecting branes and describe how to construct generic solutions. We consider the following bosonic sector of a low energy effective action of superstring theory or M-theory in D dimensions ($D \leqslant 11$):

$$S = \frac{1}{16\pi G_D} \int \mathrm{d}^D X \sqrt{-g} \left[\mathcal{R} - \frac{1}{2}(\nabla\varphi)^2 - \sum_A \frac{1}{2 \cdot n_A!} \mathrm{e}^{a_A \varphi} F_{\mathbf{n_A}}^2 \right] - (\mathrm{C.S}), \tag{1}$$

where $\mathcal{R}$ is the Ricci scalar of a spacetime metric $g_{\mu\nu}$, $F_{\mathbf{n_A}}$ is the field strength of an arbitrary form with a degree $n_A (\leqslant D/2)$ and a_A is its coupling constant with a dilaton field φ. Each index A describes a different type of brane. The term of (C.S) is the Chern–Simon term, thus we find the Chern–Simon term in the M-theory as

$$(\mathrm{CS})_M = \frac{1}{4! \cdot 4\pi G_{11}} \int C_3 \wedge F_4 \wedge F_4, \tag{2}$$

where C_3 is a 3-form gauge field and F_4 is a field strength of the 3-form gauge field. Although we leave the spacetime dimension D free, the present action is most suitable for describing the bosonic part of $D = 10$ or $D = 11$ supergravity.

As for a metric form for a spacetime with intersecting branes, we assume the following metric form:

$$ds^2 = -e^{2\xi}\,(dt + \mathcal{A})^2 + \sum_{i=1}^{d-1} e^{2\eta}\,dx_i^2 + \sum_{\alpha=1}^{p} e^{2\zeta_\alpha}\,dy_\alpha^2, \tag{3}$$

where $D = d + p$ and the dual basis $\theta^{\hat{A}}$ is given by

$$\theta^{\hat{t}} = e^{\xi}(dt + \mathcal{A}), \qquad \theta^{\hat{i}} = e^{\eta}\,dx^i, \qquad \theta^{\hat{\alpha}} = e^{\zeta_\alpha}\,dy^\alpha. \tag{4}$$

This metric form includes rotation of spacetime. Since we are interested in a stationary solution, we assume that the metric components $f, \mathcal{A} = \mathcal{A}_i\,dx^i, \xi, \eta$ and ζ_α depend only on the spatial coordinates x^i in d-dimensions, which are given by $\{t, x^i (i = 1, 2, \ldots, d-1)\}$. In this setting, we set each brane A in a submanifold of p-spatial dimensions, whose coordinates are given by $\{y_\alpha(\alpha = 1, 2, \ldots, p)\}$. Note that the solution in this metric form is invariant under the gauge transformation, $\mathcal{A} \to \mathcal{A} + d\Lambda, t \to t - \Lambda$.

As for the n_A-form field with a q_A-brane, we assume that the source brane exists in the coordinates $\left\{y_1, \ldots, y_{\alpha_{q_A}}\right\}$. The form field generated by an 'electric' charge is given by the following form:

$$F_{\mathbf{n_A}} = \partial_j E_A\,dx^j \wedge dt \wedge dy_1 \wedge \cdots \wedge dy_{q_A} + \partial_i B_j^A\,dx^i \wedge dx^j \wedge dy_1 \wedge \cdots \wedge dy_{q_A}, \tag{5}$$

where $n_A = q_A + 2$ and E_A and B_j^A are scalar and vector potentials. This setting automatically guarantees the Bianchi identity.

We can also discuss the form field generated by a 'magnetic' charge by use of a dual $^*n_A$-field with $^*q_A$-brane, which is obtained by a dual transformation of the n_A-field with a q_A-brane ($^*n_A \equiv D - n_A, {}^*q_A \equiv {}^*n_A - 2$). In other words, the field components of $F_{\mathbf{n_A}}$ generated by a 'magnetic' charge are described by the same form of (5) of the dual field $^*F_{\mathbf{n}_A} = F_{^*n_A}$. We then treat $F_{^*n_A}$, which is generated by a 'magnetic' charge, as another independent form field with a different brane from $F_{\mathbf{n_A}}$, which is generated by an 'electric' charge, when we sum up by the types of branes A.

We obtain our metric in D-dimensions as

$$ds_D^2 = \prod_A H_A^{\frac{q_A+1}{\Delta_A}} \left[-\prod_B H_B^{-2\frac{D-2}{\Delta_B}}\,(dt + \mathcal{A})^2 + \sum_{i=1}^{d-1} dx_i^2 + \sum_{\alpha=1}^{p} \prod_A H_A^{-2\frac{\gamma_{\alpha A}}{\Delta_A}}\,dy_\alpha^2 \right]. \tag{6}$$

where

$$\begin{aligned} \Delta_A &= (q_A + 1)(D - q_A - 3) + \frac{D-2}{2} a_A^2, \\ \gamma_{\alpha A} &= \delta_{\alpha A} + q_A + 1 = \begin{cases} D-2 & \alpha = \alpha_2, \ldots, \alpha_{q_A} \\ 0 & \text{otherwise} \end{cases}. \end{aligned} \tag{7}$$

3. 'Black' brane solutions with M2–M2–M2 branes: the case of $d = 5$

We consider solutions in five dimensions via torus compactification. There are three M2-branes. The metric in five dimensions is written by

$$d\bar{s}_5^2 = -\Xi^2 \left(dt + \frac{\mathcal{A}}{2} \right)^2 + \Xi^{-1}\,ds_{\mathbb{E}^4}^2, \tag{8}$$

where $\Xi = [H_2 H_5(1+f)]^{-1/3}$. The unknown functions $H_A (A = 2, 5)$, $\mathcal{A}_i$ and f satisfy the following equations:

$$\mathcal{F}^{(A)}_{ij} \equiv 2H_A \left(\mathcal{A}_{[i}\partial_{j]}E_A + \partial_{[i}B^A_{j]}\right) = \mathcal{F}_{ij} + H_A q^{(A)}_{ij} \tag{9}$$

$$\partial_j q^{(A)ij} = \partial_j \mathcal{F}^{ij} = 0 \tag{10}$$

$$\partial^2 H_A = \tfrac{1}{2} q^{(B)}_{ij} q^{(C)ij} \tag{11}$$

where $\mathcal{F}_{ij} = \partial_i \mathcal{A}_j - \partial_j \mathcal{A}_i$, and we get a constraint about H_A as $Q_{ij} = \sum H_X q^{(X)}_{ij} = 0$.

In what follows, adopting the hyperspherical coordinates as a curvilinear coordinate system, we show explicitly how to construct the exact solutions.

Now we look out the supersymmetric condition, which can be shown by the Killing spinor equation for the Majorana spinor ϵ as

$$\delta\psi_a = \left[e^{\mu}{}_a\partial_\mu + \frac{1}{4}\omega^{bc}{}_a\gamma_{bc} + \frac{1}{288}\left(\gamma_a{}^{bcdf} - 8\delta^b_a\gamma^{cdf}\right)F_{bcdf}\right]\epsilon = 0. \tag{12}$$

Then we must choose the chiral condition

$$\gamma^{t\alpha_1\alpha_2} = \gamma^{t\alpha_3\alpha_4} = \gamma^{t\alpha_5\alpha_6} = 1, \tag{13}$$

and the anti-self-dual for the deference between the $\mathcal{F}_{ij}$ and $\mathcal{F}^{(A)}_{ij}$, i.e.,

$$\mathcal{F}^{(A)}_{ij} = \mathcal{F}_{ij} + H_A q^{(A)}_{ij}, \tag{14}$$

where $\mathcal{F}^{(A)}_{ij}$ is a self-dual and $q^{(A)}_{ij}$ is an anti-self-dual. Also $q^{(A)}_{ij}$ must satisfy the

$$Q_{ij} = \sum_A H_A q^{(A)}_{ij} = 0, \tag{15}$$

or we must choose $\mathcal{F}_{ij} = \mathcal{F}^{(A)}_{ij}$ where $\mathcal{F}^{(A)}_{ij}$ is self-dual.

4. Hyperpolorical coordinates

Our next example is the hyperpolorical coordinates (ξ, η, ϕ, ψ), which are defined by the transformation

$$x_1 + \mathrm{i}x_2 = \frac{R\sinh\xi}{\cosh\xi - \cos\eta}\,\mathrm{e}^{\mathrm{i}\psi}, \qquad x_3 + \mathrm{i}x_4 = \frac{R\sin\eta}{\cosh\xi - \cos\eta}\,\mathrm{e}^{\mathrm{i}\phi}, \tag{16}$$

where $\xi \geqslant 0$, $0 \leqslant \eta \leqslant \pi$, and $0 \leqslant \phi, \psi \leqslant 2\pi$. This coordinates could be used to describe a ring topology. In this case, the infinity corresponds to $\xi = 0$, which also describes one of the symmetric axes.

The line element is given by

$$\mathrm{d}s^2_{\mathbb{E}^4} = \frac{R^2}{(\cosh\xi - \cos\eta)^2}(\mathrm{d}\xi^2 + \sinh^2\xi\,\mathrm{d}\psi^2 + \mathrm{d}\eta^2 + \sin^2\eta\,\mathrm{d}\phi^2). \tag{17}$$

We can find the general solution by using the hypergeometric function, but it is too complicated. Thus we show the typical case with the BPS sate $-q^A_\phi = q^A_\psi = q^A$. The case is already given by [29], named supersymmetric black ring solution, as

$$H_A = 1 + (\cosh\xi - \cos\eta)\left[\frac{Q_A - q^B q^C}{2R^2} + \frac{q^B q^C}{4R^2}(\cosh\xi + \cos\eta)\right], \tag{18}$$

where we assume the regularity on the symmetric axis. Q_A is an arbitrary constant.

The spacial solutions for the rotating terms of the metric are

$$\mathcal{A}_\phi = -\frac{\sin^2\eta}{8R^2}\left[\sum_X Q_X q^X - q^A q^B q^C (3 - \cosh\xi - \cos\eta)\right] \tag{19}$$

$$\mathcal{A}_\psi = -\frac{1}{2}\sum_A q^A(\cosh\xi - 1) - \frac{\sinh^2\xi}{8R^2}\left[\sum_X Q_X q^X - q^A q^B q^C (3 - \cosh\xi - \cos\eta)\right]. \tag{20}$$

This is a solution of supersymmetric black ring solutions which was introduced by Elvang *et al* ([29]).

However we consider the condition of $Q_{ij} = 0$, $\mathcal{A}_\phi$ and $\mathcal{A}_\psi$ are vanishing. Non-rotating black ring solution has a deficit angle, thus this solution is not a black hole solution.

5. Concluding remarks

In this paper, we have studied a stationary 'black' brane solution in M/superstring theory. Assuming a BPS-type relation between the first-order derivatives of metric functions, we have shown how to construct a stationary 'black' brane solution with a time-like Killing vector.

Using the hyperbipolor coordinate system, we present exact solutions in 11-dimensional M theory for the case with M2⊥ M2⊥ M2 intersecting branes with Chern–Simons terms. Compactifying these solutions into five dimensions, we show that these solutions include the supersymmetric black ring solutions, but there is another constraint for the supersymmetry the angular momentum of black ring is banishing.

Although we assume the BPS-type relations for the metric, we have to solve the elliptic-type differential equations if we want to find most general solutions, especially non-BPS spacetimes. For this purpose, we need a completely different approach such as a soliton technique to generate new solutions [34].

We have found the BPS and non-BPS rotating asymptotically flat stringy black holes, from which we may learn more about connections between microscopic and macroscopic states of gravitating objects. In our framework, we consider a toroidally compactified string theory, but one may embed the BMPV type geometry in M-theory compactified on generic Calabi–Yau spaces, which would be more interesting.

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